**Overview:** The overview statement is intended to provide a summary of major themes in this unit.

This unit extends knowledge of numerical and algebraic expressions and equations from previous grades, and it develops understanding of properties of integer exponents, square and cube roots, integer powers of 10, and scientific notation.

**Teacher Notes:** The information in this component provides additional insights which will help educators in the planning process for this unit.

- Students should extend their knowledge of whole number exponential notation with powers of 10 to negative integer notation.
- Models should be used to demonstrate the properties of integer exponents, for example models that use the Pythagorean Theorem.
- Students should understand the usefulness of integer exponential notation with powers of 10 to express very large or very small values.

**Enduring Understandings:** Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

At the completion of the unit on Expressions and Equations, the student will understand that:

- Properties of integer exponents can be used to generate equivalent numerical expressions.
- Just as numbers can be squared, cubed, and raised to the nth power, the root values (i.e., square root, cube root, nth root, etc.) of numbers can be determined or approximated.
- The square roots of rational numbers are not always rational, but sometimes are irrational.

**Essential Question(s):** A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

When are radicals and integer exponents used in expressions and equations to tell a story or represent a situation in life?

**Content Emphasis by Cluster in Grade 8:** According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), some clusters require greater emphasis than others. The list below shows PARCC’s relative emphasis for each cluster. Prioritization does not imply neglect or exclusion of material. Clear priorities are intended to ensure that the relative importance of content is properly attended to. Note that the prioritization is stated in terms of cluster headings.

- **Key:** ■ Major Clusters ◇ Supporting Clusters ☕ Additional Clusters

**The Number System**

- □ Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability
- Investigate patterns of association in bivariate data.

Focus Standard (Listed as Examples of Opportunities for In-Depth Focus in the PARCC Content Framework document): According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), this component highlights some individual standards that play an important role in the content of this unit. Educators may choose to give the indicated mathematics an especially in-depth treatment, as measured for example by the number of days; the quality of classroom activities for exploration and reasoning; the amount of student practice; and the rigor of expectations for depth of understanding or mastery of skills.

- 8.EE.7 Work with radicals and integer exponents contributes to this culminating standard for solving one-variable linear equations.
- 8.EE.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

Possible Student Outcomes: The following list is meant to provide a number of achievable outcomes that apply to the lessons in this unit. The list does not include all possible student outcomes for this unit, nor is it intended to suggest sequence or timing. These outcomes should depict the content segments into which a teacher might elect to break a given standard. They may represent groups of standards that can be taught together.
Draft Unit Plan: Grade 8 – Expressions and Equations

The student will:
- Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- Use the radical symbol to determine the relationships of square roots with squares and of cube roots with cubes.
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small values.
- Perform operations with numbers expressed in scientific notation.
- Approximate and compare the value of irrational numbers using rational numbers.

Progressions from Common Core State Standards in Mathematics: For an in-depth discussion of the overarching, “big picture” perspective on student learning of content related to this unit, see:


Vertical Alignment: Vertical curriculum alignment provides two pieces of information: (1) a description of prior learning that should support the learning of the concepts in this unit, and (2) a description of how the concepts studies in this unit will support the learning of additional mathematics.

- **Key Advances from Previous Grades:** Students enlarge their concept of and ability to compute with rational numbers by:
  - understanding of and skill with multiplication, division, decimals, and fractions
  - using properties of operations systematically to work with variables, variable expressions, and equations
  - working with the system of rational numbers to include using positive and negative numbers to describe quantities
  - extending the number line and coordinate plane to represent rational numbers and ordered pairs
  - understanding ordering and absolute value of rational numbers

- **Additional Mathematics:** Students will use skills with radicals and integer exponents:
  - in algebra when taking square roots, completing the square, using the quadratic formula, and factoring
  - in algebra when extending polynomial identities to the complex numbers
  - in geometry when proving polynomial identities to generate Pythagorean triplets; when working with circles
  - in trigonometry when studying sine, cosine, tangent, and other trigonometric ratios
  - in calculus when working with non-linear functions (i.e., quadratic, exponential, logarithmic, etc.)
  - in statistics when describing statistical data

Possible Organization of Unit Standards: This table identifies additional grade-level standards within a given cluster that support the over-arching unit standards from within the same cluster. The table also provides instructional connections to grade-level standards from outside the cluster.
## Draft Unit Plan: Grade 8 – Expressions and Equations

<table>
<thead>
<tr>
<th>Over-Arching Unit Standards</th>
<th>Supporting Standards within the Cluster</th>
<th>Instructional Connections outside the Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.EE.1:</strong> Know and apply the <strong>properties of integer exponents</strong> to generate equivalent numerical expressions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8.EE.2:</strong> Use square root and cube root symbols to represent solutions to equations in the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive <strong>rational number</strong>. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is <strong>irrational</strong>.</td>
<td></td>
<td><strong>8.NS.1:</strong> Know that numbers that are not <strong>rational</strong> are called <strong>irrational</strong>. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. <strong>8.NS.2:</strong> Use <strong>rational</strong> approximations of irrational numbers to compare the size of <strong>irrational</strong> numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). <strong>8.G.6:</strong> Explain a <strong>proof of the Pythagorean Theorem</strong> and its <strong>converse</strong>. <strong>8.G.7:</strong> Apply the <strong>Pythagorean Theorem</strong> to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. <strong>8.G.8:</strong> Apply the <strong>Pythagorean Theorem</strong> to find the distance between two points in a coordinate system. <strong>8.G.9:</strong> Know the formulas for the <strong>volumes of cones, cylinders</strong>, and <strong>spheres</strong> and use them to solve real-world and mathematical problems.</td>
</tr>
<tr>
<td><strong>8.EE.3:</strong> Use numbers expressed in the form of a single digit times an</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over-Arching Unit Standards</td>
<td>Supporting Standards within the Cluster</td>
<td>Instructional Connections outside the Cluster</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------------------------------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.</td>
<td>8.EE.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.</td>
<td></td>
</tr>
</tbody>
</table>

Connections to the Standards for Mathematical Practice: This section provides samples of how the learning experiences for this unit support the development of the proficiencies described in the Standards for Mathematical Practice. The statements provided offer a few examples of connections between the Standards for Mathematical Practice in the content standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

In this unit, educators should consider implementing learning experiences which provide opportunities for students to:

1. Make sense of problems and persevere in solving them.
   - Recognize that properties of integer exponents can be used to generate equivalent numerical expressions.
   - Differentiate between principal (positive) and negative roots.
   - Decide if a square root or cube root is rational or irrational.
2. **Reason abstractly and quantitatively**
   - Compare large and small numbers using integer powers of 10.
   - Perform operations with numbers expressed in scientific notation.

3. **Construct Viable Arguments and critique the reasoning of others.**
   - Recognize and justify the difference between rational numbers and irrational numbers.
   - Justify that properties of integer exponents produce equivalent numerical expressions.

4. **Model with Mathematics**
   - Use a number line diagram to estimate and locate the approximate values of irrational numbers by comparing them to the size of rational numbers.
   - Construct a nonverbal representation of a verbal problem.
   - Construct a model to demonstrate the progression of values for integer powers of 10.

5. **Use appropriate tools strategically**
   - Use technology or manipulatives to explore a problem numerically or graphically.

6. **Attend to precision**
   - Use mathematics vocabulary (i.e., whole number, rational number, irrational number, square root, cube root, principal root, negative root, etc.) properly when discussing problems.
   - Demonstrate their understanding of the mathematical processes required to solve a problem by carefully showing all of the steps in solving the problem.
   - Label final answers appropriately.

7. **Look for and make use of structure.**
   - Make observations about how scientific notation is used to express very large or very small numbers.

8. **Look for and express regularity in reasoning**
   - Pay special attention to $10^0$

**Content Standards with Essential Skills and Knowledge Statements and Clarifications:** The Content Standards and Essential Skills and Knowledge statements shown in this section come directly from the Maryland State Common Core Curriculum Frameworks. Clarifications were added as needed. Please note that only the standards or portions of standards that needed further explanation have supporting statements. Educators should be cautioned against perceiving this as a checklist. All information added is intended to help the reader gain a better understanding of the standards.
### Standard: 8.EE.1

**Know and apply the properties of integer exponents to generate equivalent numerical expressions.**

**Essential Skills and Knowledge**
- Ability to recognize and apply the following properties of integer exponents:
  - Product/Quotient of Powers
  - Negative Exponents
  - Zero Exponents
  - Power of Powers
- Ability to apply a combination of properties to show equivalency

**Clarification**

**Properties of Integer Exponents**

- **Product of powers:** Add the exponents \( x^5 \cdot x^3 = x^8 \)
- **Quotient of powers:** Subtract the exponents \( \frac{x^5}{x^2} = x^3 \) OR \( x^5 \div x^3 = x^2 \)
- **Negative exponents:** \( x^{-3} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3} \) OR \( 2^{-3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8} \)
- **Zero exponents:** Using quotient of powers where exponents are subtracted \( \frac{x^5}{x^5} = x^0 \), and \( \frac{x^5}{x^5} = 1 \), therefore \( x^0 = 1 \)
- **Power of powers:** Multiply the exponents \( (x^4)^2 = (x \cdot x \cdot x \cdot x)^2 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) \). Therefore \( (x^4)^2 = x^8 \)

### Standard: 8.EE.2

**Use square root and cube root symbols to represent solutions to equations of the form \( x^2 = p \) and \( x^3 = p \), where \( p \) is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that \( \sqrt{2} \) is irrational.**

**Essential Skills and Knowledge**
- Ability to recognize and apply the following:
  - Perfect Squares
  - Perfect Cubes
  - Square Roots (Symbol Notation)
  - Cube Roots (Symbol Notation)
  - Principal (positive) roots/negative roots

**Clarification**

- **Perfect square:** The product of another number multiplied by itself. Examples: \( \pm 5 \cdot \pm 5 = 25 \), therefore 25 is a perfect square of 5 or \( \mp 5 \)
- **Perfect cube:** The product of another number multiplied by itself twice. Examples: \( 2 \cdot 2 \cdot 2 = 8 \), therefore 8 is a perfect cube of 2

- \( \sqrt{\frac{3}{4}} \cdot \sqrt{\frac{3}{4}} = \frac{27}{64} \), therefore \( \frac{27}{64} \) is a perfect square of \( \frac{3}{4} \)

**Square root symbol:** \( \sqrt{x} \) **Cube root symbol:** \( \sqrt[3]{x} \)

### Standard: 8.EE.3

**Use numbers expressed in the form of a single digit times an integer powers of 10 to compare large and small numbers.**

**Essential Skills and Knowledge**
- Ability to compare large and small numbers

**Clarification**

- **Integer powers of 10:** They can be positive or negative.
  - Positive examples: \( 10^1 = 10; 10^2 = 100; 10^3 = 1,000; 10^4 = \)
### Essentials and Knowledge

<table>
<thead>
<tr>
<th><strong>integer power of 10</strong></th>
<th><strong>small numbers using properties of integer exponents (see 8.EE.1)</strong></th>
<th><strong>10,000; 10^5 = 100,000</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>to estimate very large or very small quantities, and to express how many times as much one is than the other.</td>
<td>negative examples: (10^{-1} = \frac{1}{10} ); (10^{-2} = \frac{1}{100} ); (10^{-3} = \frac{1}{1000} ); (10^{-4} = \frac{1}{10,000} ).</td>
<td>(10^{-5} = \frac{1}{100,000} )</td>
</tr>
</tbody>
</table>

**8.EE.4:** Perform operations with numbers expressed in **scientific notation**, including problems where both **decimal** and scientific notation are used. Use **scientific notation** and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

**Essential Skills and Knowledge**
- Ability to compare units of measure
- Ability to read scientific notation on a calculator

**scientific notation:** A number in scientific notation is written as the product of two factors. The first factor is a number greater than or equal to 1 and less than 10; the second factor is an integer power of 10. In scientific notation 37,482,000 is written \(3.7482 \times 10^7\). Also 0.000000374 is written \(3.74 \times 10^{-7}\).

**decimal notation:** Notation refers to symbols that denote quantities and operations. Values written in decimal notation use a decimal point to differentiate between whole number values and mixed number values, for example 132 versus 132.5. Mixed number values also can be written in fraction notation, for example \(132\frac{1}{2}\).

### Evidence of Student Learning:

The Partnership for Assessment of Readiness for College and Careers (PARCC) has awarded the Dana Center a grant to develop the information for this component. This information will be provided at a later date. The Dana Center, located at the University of Texas in Austin, encourages high academic standards in mathematics by working in partnership with local, state, and national education entities. Educators at the Center collaborate with their partners to help school systems nurture students' intellectual passions. The Center advocates for every student leaving school prepared for success in postsecondary education and in the contemporary workplace.

### Fluency Expectations:

This section highlights individual standards that set expectations for fluency, or that otherwise represent culminating masteries. These standards highlight the need to provide sufficient supports and opportunities for practice to help students meet these expectations. Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient practice.
thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.

- PARCC has no fluency expectations related to work with radicals and integer exponents.

**Common Misconceptions:** This list includes general misunderstandings and issues that frequently hinder student mastery of concepts in this unit.

Students may:
- Confuse the operations for the properties of integer exponents
- Miscount the decimal places for a value in decimal notation when expressing the same value in scientific notation
- Forget that each rational number has a negative square root, as well as a principal (positive) square root

**Interdisciplinary Connections:** Interdisciplinary connections fall into a number of related categories:
- Literacy standards within the Maryland Common Core State Curriculum
- Science, Technology, Engineering, and Mathematics standards
- Instructional connections to mathematics that will be established by local school systems, and will reflect their specific grade-level coursework in other content areas, such as English language arts, reading, science, social studies, world languages, physical education, and fine arts, among others.

**Model Lesson Plans:** Note: This is just the start to the lesson plan. The following language/directions need to be cleaned up to include optional teacher questions, answers, look-fors, and student materials/worksheets.

<table>
<thead>
<tr>
<th><strong>Background Information</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Content/Grade Level</strong></td>
</tr>
<tr>
<td><strong>Unit</strong></td>
</tr>
<tr>
<td><strong>Essential Questions/Enduring Understandings Addressed in the Lesson</strong></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### Draft Unit Plan: Grade 8 – Expressions and Equations

<table>
<thead>
<tr>
<th>Enduring Understanding:</th>
<th>Very large and very small values, as found in many real-world contexts, can be expressed using powers of ten.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Standards Addressed in This Lesson</th>
<th>8.EE.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Lesson Topic</th>
<th>Powers of 10</th>
</tr>
</thead>
</table>

| Relevance/Connections | Ability to compare large and small numbers using properties of integer exponents (see 8.EE.1) Working with scientific notation (8.EE.4) presents opportunities for strategically using appropriate tools (MP.5). For example, a computation such as \((1.73 \times 10^{-4}) \times (1.73 \times 10^{-4})\) can be done quickly with a calculator by simply squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection. |

<table>
<thead>
<tr>
<th>Student Outcomes</th>
<th>(\text{multiplied by}) Students will be able to use single digits a power of 10 in order to estimate and compare very large and very small quantities.</th>
</tr>
</thead>
</table>

| Prior Knowledge Needed to Support This Learning | Prior knowledge of place value, number sense and repeated multiplication. Ability to write numbers in expanded form as powers of 10 Ability to recognize and apply the following properties of integer exponents: o Product/Quotient of Powers o Negative Exponents o Zero Exponents o Power of Powers Ability to recognize how \textit{much} greater or less than one number is compared to another (additive) Ability to recognize how \textit{many times} greater or less than one number is compared to another (multiplicative) |

<p>| Method for determining student readiness for the lesson | Use questions \textit{Determining Student Readiness for the Lesson} to assess student understanding of: Compare and order rational numbers Evaluate powers of ten (including negative exponents) Write numbers in expanded notation using powers of 10 |</p>
<table>
<thead>
<tr>
<th>Learning Experience</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Component</strong></td>
</tr>
<tr>
<td><strong>Details</strong></td>
</tr>
<tr>
<td><strong>Which Standards of Mathematical Practice does this component address?</strong></td>
</tr>
</tbody>
</table>

### Warm Up/Drill
Use properties of integer exponents to show why any base to the power of zero equals one \( (x^0 = 1) \).

“The answer to a question is 1, what is the question?”

Provide 3 or more products or quotients of powers that result in powers of 1.

Possible student responses could include:

\[
(x^5 \div x^5), \quad (7^8 \div 7^8), \quad (7^8 \cdot 7^{-8}), \quad (x^5 \cdot x^{-5})
\]

### Motivation

- Provide time for the students to explore the following website: [http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/](http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/)
  - or a similar website that shows: The Milky Way at 10 million light years from the Earth and moves through space towards the Earth in successive orders of magnitude. After that, move from the actual size of a leaf into a microscopic world.
  - After providing independent exploration time, ask the students to write about what they are viewing at each of the following powers of ten:
  - Reason abstractly and quantitatively
  - Use appropriate tools strategically
## Draft Unit Plan: Grade 8 – Expressions and Equations

### Learning Experience

<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
<th>Which Standards of Mathematical Practice does this component address?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>○ $10^0$, $10^5$, $10^0$, $10^{-5}$, $10^{-10}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>● Ask these key questions:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>○ Why do you think that the different views happen with each of the changes to the exponent?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>○ Why does the zoom get bigger or smaller?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>● On a subsequent day, use the following websites to reinforce the powers of ten concept. <a href="http://www.powersof10.com/">http://www.powersof10.com/</a></td>
<td></td>
</tr>
</tbody>
</table>

### Activity 1

**Represent powers of 10 symbolically: $10^0$, $10^5$**

- Complete **Activity #1 Chart**, using the website as a reference tool.
- Challenge students to notice patterns as the exponents change.
- Culminating question: What connection is there between the powers of ten and place value?
- Summary: Possible examples of explanations:
  - Number line
  - Place value chart
  - Verbal explanation

- Model with Mathematics
- Look for and express regularity in reasoning
<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
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</tr>
</thead>
</table>
| **Activity 2**  | **Represent powers of 10 symbolically:** $x \cdot 10^y$                | • Model with Mathematics  
• Use appropriate tools strategically  
• Attend to precision                                                                                                                |
| UDL Components  | • Multiple Means of Representation  
• Multiple Means for Action and Expression  
• Multiple Means for Engagement                                                                                                        |
| Key Questions   | • Referencing the video, provide a situation that there is a plane flying at $10^6$ feet high in the sky and a satellite $10^7$ feet high in the sky, and you are in between them.  
• Ask the students to:  
  o Explain what they see  
  o What type of vehicle they are in  
  o Put students in small groups. Pass out the Activity #2 Cards and have students determine whether the height is between the two given values. Say, “There are going to be other objects in between these two heights than you. Each group has been given a height of an object in the air. Is each object going to be in between these two heights?” Students will place in column whether “between,” “above,” or “below.”  
• How do you represent a number in exponential form that is in between these two heights?  
• Use these prompting questions to introduce numbers expressed in the form of a single digit times an integer power of 10  
• Summary→ The plane and satellite have changed their heights. The plane is now at $10^3$ and the satellite is now $10^8$. Using Activity #2 Summary Template, create two heights below, two heights between, and two heights above the objects. |
| Formative Assessment |                                                                                       |                                                                                                                                     |
| Summary         |                                                                                       |                                                                                                                                     |
| **Activity 3**  | **Provide students with real-life data points that they will convert into a whole number times a power of ten.**                      | • Make sense of problems and persevere in solving them.  
• Reason abstractly and quantitatively                                                                                               |
<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
<th>Which Standards of Mathematical Practice does this component address?</th>
</tr>
</thead>
<tbody>
<tr>
<td>UDL Components</td>
<td>Present the following data to the students: the number of text messages in America in June 2006 was 33,500,000. In June 2011 there were 196,900,000,000 text messages sent. <a href="http://www.ctia.org/advocacy/research/index.cfm/aid/10323">http://www.ctia.org/advocacy/research/index.cfm/aid/10323</a> Have the students estimate how many more texts were sent and estimate how many times more texts were sent in 2011 than 2006. Ask: how did you determine your answer? Anticipate your students using standard form instead of exponential form. Prompt a discussion about the need to convert numbers into a power of ten by asking: o How can these numbers be expressed in a different format? o Is there a format that would make these numbers easier to compare? o Demonstrate how to express numbers as a single digit times a power of ten. o Model how to compare text message values written in the form of a factor times a power of ten. Using the website, <a href="http://www.worldatlas.com/aatlas/populations/ctypopls.htm">http://www.worldatlas.com/aatlas/populations/ctypopls.htm</a> provide students with populations from the largest country (China) and the smallest country (Vatican City). Students will translate the populations as an integer times a power of 10. Use this form to compare the populations, estimating how many times larger China’s population is. Select some countries that will require rounding so that it will be a single number times a power of ten and compare the populations, estimating how many times larger one is to the other population.</td>
<td>Model with Mathematics Use appropriate tools strategically</td>
</tr>
</tbody>
</table>
## Learning Experience

<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
<th>Which Standards of Mathematical Practice does this component address?</th>
</tr>
</thead>
</table>
| **Activity 4** | Use the chart, *Translating Integer Exponents of Smaller Objects*.  
- You may want to have available some of the objects (or pictures) in the chart for this lesson.  
- You may choose to substitute some of the information in the chart to better accommodate your students personal interests.  
- The questions lead students to determine/estimate the number of cells in a 100-pound student.  
- While completing the chart activity, challenge students to identify patterns as exponents change.  
- As an extension, students can research items that represent integer exponent values not listed in the chart. (i.e.: $10^{-4}$) | - Reason abstractly and quantitatively  
- Attend to precision |
| **Closure** | Think about the size of a grain of sand. $1 \cdot 10^{-6}$ cm$^3$ Describe the process you would use to determine the weight of a bucket of sand. | - Construct Viable Arguments and critique the reasoning of others.  
- Attend to precision |

### Supporting Information

<table>
<thead>
<tr>
<th>Interventions/Enrichments</th>
</tr>
</thead>
</table>
| - Special Education/Struggling Learners  
- ELL | - ELL- Fluency with using the calculators  
- Base ten blocks to reinforce place value concepts  
- Understanding of decimal place value- provide struggling learners a desktop number line that includes powers of 10 with verbal and symbolic representations  
- Vocabulary – ELL and special education students need to know and understand the following terms to |
### Gifted and Talented
- be successful with these activities: *quotient, product, exponents, integer, power of powers, decimal notation,* and *standard form.*
- Enrichment – Have students investigate and identify tools that are used to measure the length, mass, and weight of small items.

### Materials
- **Questions** *Determining Student Readiness for the Lesson*
- **Activity #1 Chart**
- **Activity #2 Cards**
- **Activity #2 Summary Template**
- **Translating Integer Exponents of Smaller Objects** for Activity 4, including objects/pictures

### Technology
- Computer with internet access
- LCD projector
- Calculators

### Resources
**(must be available to all stakeholders)**
- [http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_05(extra_examples)/chapter8/lesson8_3.pdf](http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_05(extra_examples)/chapter8/lesson8_3.pdf)
- [http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm](http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm)
- [http://www.worldatlas.com/aatlas/populations/ctypopls.htm](http://www.worldatlas.com/aatlas/populations/ctypopls.htm)
Determining Student Readiness for the Lesson

1. Look at these four rational numbers.

\[
0.875 \quad \frac{-2}{3} \quad \frac{11}{12} \quad -42
\]

Compare the values of these four numbers and order them from least to greatest.

2. Evaluate these powers of 10:

\[
10^7 \quad 10^1 \quad 10^{-4} \quad 10^0
\]

3. Write these numbers in expanded notation using powers of 10.

\[
678.00042 \quad 23,000,150 \quad 9.00009
\]
### Activity #1 Chart

<table>
<thead>
<tr>
<th>Power of Ten</th>
<th>Standard Form</th>
<th>What do you see?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^x$ (you select the $x$ value)</td>
<td></td>
<td></td>
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<tr>
<td>$10^5$</td>
<td></td>
<td></td>
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<td>$10^4$</td>
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<td>$10^1$</td>
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<td>$10^0$</td>
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<td>$10^{-1}$</td>
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<td></td>
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<tr>
<td>$10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-x}$ (you select the $x$ value)</td>
<td></td>
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</table>
### Activity #2 Cards

<table>
<thead>
<tr>
<th>Object A</th>
<th>Object B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^5$</td>
<td>$13 \times 10^6$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Object C</th>
<th>Object D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^{-6}$</td>
<td>$3 \times 10^7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Object E</th>
<th>Object F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$13 \times 10^5$</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>Object G</td>
<td>Object H</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>$5 \times 10^6$</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>Object I</td>
<td>Object J</td>
</tr>
<tr>
<td>$1,255,032 \times 10^0$</td>
<td>$126 \times 10^4$</td>
</tr>
</tbody>
</table>
## Activity #2 Summary Template

<table>
<thead>
<tr>
<th></th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>___ X 10^4</td>
<td></td>
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<tr>
<td>___ X 10^3</td>
<td></td>
</tr>
<tr>
<td>Plane 10^3</td>
<td></td>
</tr>
<tr>
<td>___ X 10^2</td>
<td></td>
</tr>
<tr>
<td>___ X 10^1</td>
<td></td>
</tr>
<tr>
<td>Satellite 10^4</td>
<td></td>
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<tr>
<td>___ X 10</td>
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<td>___ X 10</td>
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<tr>
<td></td>
<td>Explanation</td>
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<tr>
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<td>-------------</td>
</tr>
<tr>
<td>___ X 10^0</td>
<td></td>
</tr>
<tr>
<td>___ X 10^1</td>
<td></td>
</tr>
<tr>
<td><strong>Plane</strong> 10^3</td>
<td></td>
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<tr>
<td>___ X 10^2</td>
<td></td>
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<tr>
<td>___ X 10^3</td>
<td></td>
</tr>
<tr>
<td><strong>Satellite</strong> 10^4</td>
<td></td>
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<tr>
<td>___ X 10^0</td>
<td></td>
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<tr>
<td>___ X 10^1</td>
<td></td>
</tr>
<tr>
<td>___ X 10^2</td>
<td></td>
</tr>
</tbody>
</table>
Activity #4

Chart:
## Translating Integer Exponents of Smaller Objects

<table>
<thead>
<tr>
<th>Object</th>
<th>100 Pound Student</th>
<th>Jar of Peanut Butter</th>
<th>2 Large Candy Bars</th>
<th>Bite Size Candy Bar</th>
<th>1 Chocolate Chip Cookie</th>
<th>5 Mustard Seeds</th>
<th>1 Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass in g</td>
<td>45360 g</td>
<td>1000g</td>
<td>100g</td>
<td>10g</td>
<td>1g</td>
<td>.01g</td>
<td></td>
</tr>
<tr>
<td>Mass in kg</td>
<td>45.36 kg</td>
<td>1kg</td>
<td>1/10 kg</td>
<td>1/100 kg</td>
<td>1/1000kg</td>
<td>1/10000kg</td>
<td></td>
</tr>
<tr>
<td>Mass in Power of ten (kg)</td>
<td>$10^0$</td>
<td>$10^1$</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
<td>$10^{-5}$</td>
<td>$10^{-12}$</td>
<td></td>
</tr>
</tbody>
</table>

If you were to make a peanut butter model of yourself, how many cells would it be?

If you were to make a chocolate chip model of yourself, how many cells would it be?

How many cells would there be in 100 pound student?
Lesson Seeds: The lesson seeds have been written particularly for the unit, with specific standards in mind. The suggested activities are not intended to be prescriptive, exhaustive, or sequential. Rather, they simply demonstrate how specific content can be used to help students learn the skills described in the standards. They are designed to generate evidence of student understanding and give teachers ideas for developing their own activities.

MSDE Mathematics Lesson Seed Organizer

<table>
<thead>
<tr>
<th>Domain</th>
<th>Standard : 8.EE.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purpose/Big Idea:</td>
<td>Generating equivalent numerical expressions with integer exponents.</td>
</tr>
<tr>
<td>Materials:</td>
<td>Power point of expressions; Bingo cards.</td>
</tr>
<tr>
<td>Activity:</td>
<td>Practice simplifying one-term expressions with integer exponents through a Bingo game. Give each student a Bingo card. Tell students that you will give them a problem and they work it out and look for the answer on their card. From there they cross out answers that they find. If they get 5 in a row, column or diagonal, they get Bingo. Run power point or display problems one at a time in whatever method you choose. (Cards are attached.) (This activity comes from Dan Meyer, blog.mrmeyer.com/)</td>
</tr>
<tr>
<td>Essential Question:</td>
<td>When are radicals and integer exponents used in expressions and equations to tell a story or represent a situation in life?</td>
</tr>
</tbody>
</table>
Bingo

\[
\begin{align*}
(x^7)^3 (3x^2)^4 &\quad \frac{1}{x^2 \cdot x^{-5}} \\
(d^4)^2 (d^7)^0 &\quad b^{-2} \cdot b^4 \cdot b \\
(3x^7)^2 &\quad x^6 \cdot y^{-2} \cdot x^4 \\
(2x^3)^2 &\quad d^2 \cdot d^{-7} \\
a^5 \cdot 3b^7 \cdot 2a^{-6} &\quad 5^{-13} \cdot 5^5 \cdot 5^2 \\
5d^2 (d^7)^2 &\quad d^0 \cdot (d^7)^0 \\
(3x^7)^3 &\quad (4a^3 b^{-2})^{-2}
\end{align*}
\]
Bingo

\[(d^8)^4\]
\[(x^{-2})^2(3xy^2)^4\]
\[(6mn)^3(m^{-3})^2\]
\[2^{-2} \cdot 2^3\]
\[(d^4)^7(d^2)^{-3}\]
\[(2^3)^7(2^{-4})^4\]
\[(t^2)^{-2}(4t^2)^2\]
<table>
<thead>
<tr>
<th>Term</th>
<th>6$b^7$</th>
<th>5</th>
<th>$27x^{21}$</th>
<th>$b^3$</th>
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<tbody>
<tr>
<td>$\frac{1}{a}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{d^{12}}$</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>$32$</td>
<td>16</td>
<td>$d^8$</td>
<td>$\frac{b^4}{16a^6}$</td>
<td>$\frac{1}{d^5}$</td>
<td></td>
</tr>
<tr>
<td>$11$</td>
<td>$4x^6$</td>
<td>FREE</td>
<td>$d^{32}$</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>$x^3$</td>
<td>10</td>
<td>$81x^{29}$</td>
<td>$\frac{x^{10}}{y^2}$</td>
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<td>$d^{22}$</td>
<td>2</td>
<td>$\frac{1}{5^6}$</td>
<td>$\frac{216n^3}{m^3}$</td>
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<tr>
<td>(\frac{6b^7}{a})</td>
<td>(\frac{5}{d^{12}})</td>
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<td>(b^3)</td>
<td>(1)</td>
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<td>$\frac{x^{10}}{y^2}$</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Sample Assessment Items: The items included in this component will be aligned to the standards in the unit and will include:

- Items purchased from vendors
- PARCC prototype items
- PARCC public release items
- Maryland Public release items

Interventions/Enrichments/PD: (Standard-specific modules that focus on student interventions/enrichments and on professional development for teachers will be included later, as available from the vendor(s) who will produce the modules.)

Vocabulary/Terminology/Concepts: This section of the Unit Plan is divided into two parts. Part I contains vocabulary and terminology from standards that comprise the cluster which is the focus of this unit plan. Part II contains vocabulary and terminology from standards outside of the focus cluster. These “outside standards” provide important instructional connections to the focus cluster.

**Part I – Focus Cluster: Expressions and Equations**

**Properties of Integer Exponents:** These properties include product of powers, quotient of powers, negative exponents, zero exponent, and power of powers.

- **Product of Powers:** Add the exponents. $x^5 \cdot x^3 = x^8$ OR $2^5 \cdot 2^3 = 2^8$
- **Quotient of Powers:** Subtract the exponents. $\frac{x^5}{x^3} = x^2$ OR $x^5 \div x^3 = x^2$ OR $\frac{2^5}{2^3} = 2^2$
- **Negative Exponents:** $x^{-3} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$ OR $2^{-3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$
- **Zero Exponents:** Using quotient of powers where exponents are subtracted. $\frac{x^5}{x^5} = x^0$, and $\frac{x^5}{x^5} = 1$, therefore $x^0 = 1$ OR $2^0 = 1$
- **Power of Powers:** Multiply the exponents. $(x^4)^2 = (x \cdot x \cdot x \cdot x)^2 = (x \cdot x \cdot x \cdot x) (x \cdot x \cdot x \cdot x)$. Therefore $(x^4)^2 = x^8$
  OR $(2^4)^2 = (2 \cdot 2 \cdot 2 \cdot 2)^2 = (2 \cdot 2 \cdot 2 \cdot 2) (2 \cdot 2 \cdot 2 \cdot 2)$. Therefore $(2^4)^2 = 2^8$

**Perfect Square:** A perfect square is the product of a number multiplied by itself.

Examples: $4 \cdot 4 = 16$, therefore 16 is a perfect square of 4;

$6 \cdot 6 = 36$, therefore 36 is a perfect square of "6;
\[
\frac{7}{10} \cdot \frac{7}{10} = \frac{49}{100}, \text{ therefore } \frac{49}{100} \text{ is a perfect square of } \frac{7}{10}
\]

**perfect cube**: A perfect cube is the product of a number multiplied by itself twice.
Examples: \(2 \cdot 2 \cdot 2 = 8\), therefore 8 is a perfect cube of 2
\[\bar{5} \cdot \bar{5} \cdot \bar{5} = \bar{125}, \text{ therefore } \bar{125} \text{ is a perfect cube of } \bar{5}\]
\[\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}, \text{ therefore } \frac{27}{64} \text{ is a perfect cube of } \frac{3}{4}\]

**square root**: The square root of a number is a value which, when used as a factor two times produces the given number. The square root symbol is \(\sqrt{\phantom{x}}\). Example: \(\sqrt{144}\) (read as square root of 144) is 12 because \(12 \cdot 12 = 144\).

**cube root**: A cube root of a number is a value which, when used as a factor three times produces the given number. The cube root symbol is \(\sqrt[3]{\phantom{x}}\). Example: \(\sqrt[3]{216}\) (read as cube root of 216) is 6 because \(6 \cdot 6 \cdot 6 = 216\).

**relationship between square/square root and cube/cube root**: For example, the square of 6 (and \(\bar{6}\)) = 36; the square root of 36 = 6 (and \(\bar{6}\)).

The cube of 5 = 125; the cube root of 125 = 5. The cube of \(\bar{3}\) = \(\bar{27}\); the cube root of \(\bar{27}\) = \(\bar{3}\).

**principal (positive) root and negative root**: A positive number has two square roots. The principal root is positive and the other root is negative.
For example the square roots of 121 are 11 (principal root) and \(\bar{11}\) (negative root) because \(11^2 = 121\) and \((\bar{11})^2 = 121\).

**integer powers of 10**: Integer powers of 10 are numbers with a base of 10 and an exponent that is an integer.
Examples of positive: \(10^1 = 10\); \(10^2 = 100\); \(10^3 = 1,000\); \(10^4 = 10,000\); \(10^5 = 100,000\)
Examples of negative: \(10^{-1} = \frac{1}{10}\); \(10^{-2} = \frac{1}{100}\); \(10^{-3} = \frac{1}{1,000}\); \(10^{-4} = \frac{1}{10,000}\); \(10^{-5} = \frac{1}{100,000}\)
Example of zero: \(10^0 = 1\)

**scientific notation**: A number in scientific notation is written as the product of two factors. The first factor is a number greater than or equal to 1 and less than 10; the second factor is an integer power of 10.
Example: \(37,482,000\) is written \(3.7482 \times 10^7\)
\(0.00000037482\) is written \(3.7482 \times 10^{-7}\)
**decimal notation:** Notation refers to symbols that denote quantities and operations. Values written in decimal notation use a decimal point to differentiate between whole number values and mixed number values, for example 132 versus 132.5. Mixed number values also can be written in fraction notation, for example \(132\frac{1}{2}\).

**Part II – Instructional Connections outside the Focus Cluster**

**rational numbers:** Numbers that can be expressed as an integer, as a quotient of integers (such as \(\frac{1}{2}, \frac{4}{3}, 7\)), or as a decimal where the decimal part is either finite or repeats infinitely (such as 2.75 and 33.3333…) are considered rational numbers.

**irrational numbers:** A number is irrational because its value cannot be written as either a finite or a repeating decimal such as \(\pi\) and \(\sqrt{2}\).

**real number system:** The set of numbers consisting of rational and irrational numbers make up the real number system.

**truncate:** In this estimation strategy, a number is shortened by dropping one or more digits after the decimal point (i.e., 234.56 is truncated to the tenth’s place \(\rightarrow 234.5\) by dropping the digit 6 in the hundredth’s place).

**proof of the Pythagorean Theorem and it converse:** In any right triangle, the sum of the squares of the legs equals the square of the hypotenuse \((\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2)\). The figure below shows the parts of a right triangle.
The distance formula: The distance \(d\) between the points \(A = (x_1, y_1)\) and \(B = (x_2, y_2)\) is given by the formula:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

This formula is an application of the Pythagorean Theorem for right triangles:
The formula for the volume of a cone can be determined from the volume formula for a cylinder. We must start with a cylinder and a cone that have equal heights and radii, as in the diagram below.
Imagine copying the cone so that we had three congruent cones, all having the same height and radii of a cylinder. Next, we could fill the cones with water. As our last step in this demonstration, we could then dump the water from the cones into the cylinder. If such an experiment were to be performed, we would find that the water level of the cylinder would perfectly fill the cylinder.

This means it takes the volume of three cones to equal one cylinder. Looking at this in reverse, each cone is one-third the volume of a cylinder. Since a cylinder’s volume formula is \( V = Bh \), then the volume of a cone is one-third that formula, or \( V = \frac{Bh}{3} \). Specifically, the cylinder’s volume formula is \( V = \pi r^2 h \) and the cone’s volume formula is \( V = \frac{\pi r^2 h}{3} \).

**Volume of Cylinders:** The process for understanding and calculating the volume of cylinders is identical to that of prisms, even though cylinders are curved. Here is a general right cylinder.
Let's start with a specific right cylinder of radius 3 units and height 4 units.

![Specific Cylinder]

We fill the bottom of the cylinder with unit cubes. This means the bottom of the prism will act as a container and will hold as many cubes as possible without stacking them on top of each other. This is what it would look like.

![Filled Cylinder]

The diagram above is strange looking because we are trying to stack cubes within a curved space. Some cubes have to be shaved so as to allow them to fit inside. Also, the cubes do not yet represent the total volume. It only represents a partial volume, but we need to count these cubes to arrive at the total volume. To count these full and partial cubes, we will use the formula for the area of a circle.
The radius of the circular base (bottom) is 3 units and the formula for area of a circle is \( A = \pi r^2 \). So, the number of cubes is \((3.14)(3)^2 = (3.14)(9)\), which to the nearest tenth, is equal to 28.3.

If we imagine the cylinder like a building (like we did for prisms above), we could stack cubes on top of each other until the cylinder is completely filled. It would be filled so that all cubes are touching each other such that no space existed between cubes. It would look like this.

To count all the cubes above, we will use the consistency of the solid to our advantage. We already know there are 28.3 cubes on the bottom level and all levels contain the exact number of cubes. Therefore, we need only take that bottom total of 28.3 and multiply it by 4 because there are four levels to the cylinder. \(28.3 \times 4 = 113.2\) total cubes to our original cylinder.

If we review our calculations, we find that the total bottom layer of cubes was found by using the area of a circle, \( \pi r^2 \). Then, we took the result and multiplied it by the cylinder's height. So the volume of a cylinder is \( \pi \) times the square of its radius times its height.

**volume of spheres**: A sphere is the locus of all points in a region that are equidistant from a point. The two-dimensional rendition of the solid is represented below.
To calculate the surface area of a sphere, we must imagine the sphere as an infinite number of pyramids whose bases rest on the surface of the sphere and extend to the sphere's center. Therefore, the radius of the sphere would be the height of each pyramid. One such pyramid is depicted below.

The volume of the sphere would then be the sum of the volumes of all the pyramids. To calculate this, we would use the formula for the volume of a pyramid, namely \( V = \frac{Bh}{3} \). We would take the sum of all the pyramid bases, multiply by their height, and divide by 3.
First, the sum of the pyramid bases would be the surface area of a sphere, \(SA = 4\pi r^2\). Second, the height of each of the pyramids is the radius of the sphere, \(r\). Third, we divide by three. The result of these three actions is volume of a sphere \(V = \frac{(4\pi r^2)(r)}{3}\) or \(V = \frac{4\pi r^3}{3}\).

**Resources:** This section contains links to materials that are intended to support content instruction in this unit.